

hw2 B09505021 Numerical Analysis

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A2.

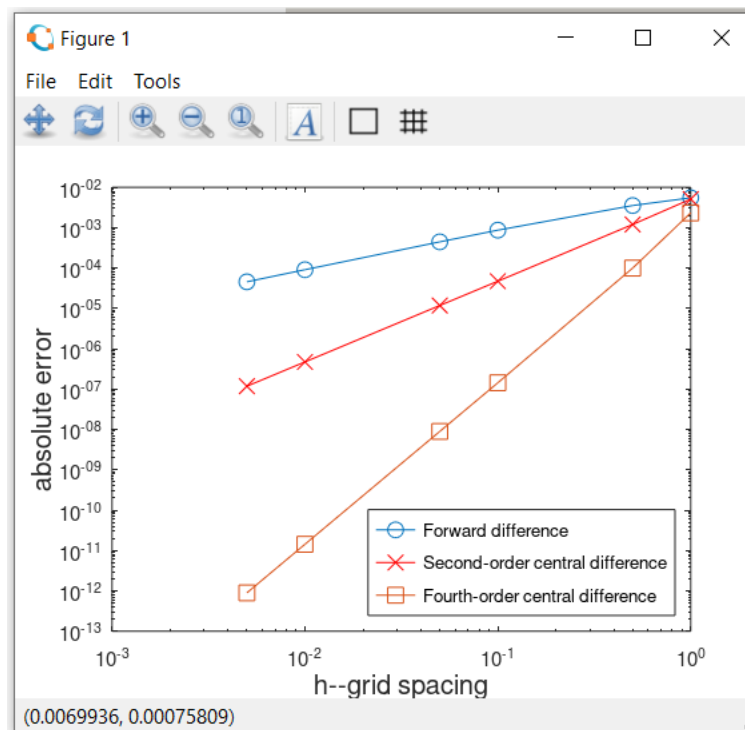
```
x = 4;
h = [1; 0.5; 0.1; 0.05; 0.01; 0.005];
n = size(h, 1)
yexact = (x*cos(x)-3*sin(x))/(x^4);
yforward = zeros(n, 1);
ysoc = zeros(n, 1);
yfoc = zeros(n, 1);

for i = 1:n
    yt1(i) = ((sin(x.+h(i))./((x.+h(i)).^3) - sin(x)./(x.^3))./h(i));
    yforward(i) = abs(yt1(i)-yexact);
endfor

for j = 1:n
    yt2(j) = ((sin(x.+h(j))./((x.+h(j)).^3) - sin(x.-h(j))./((x.-h(j)).^3))./(2*h(j)));
    ysoc(j) = abs(yt2(j)-yexact)
endfor

for k = 1:n
    yt3(k) = (8.*(sin(x.+h(k))./((x.+h(k)).^3) - 8.*(sin(x.-h(k))./((x.-h(k)).^3) - (sin(x.+2.*h(k))./((x.+2.*h(k)).^3) + (sin(x.-2.*h(k))./((x.-2.*h(k)).^3))))./(24*h(k)));
    yfoc(k) = abs(yt3(k)-yexact)
endfor

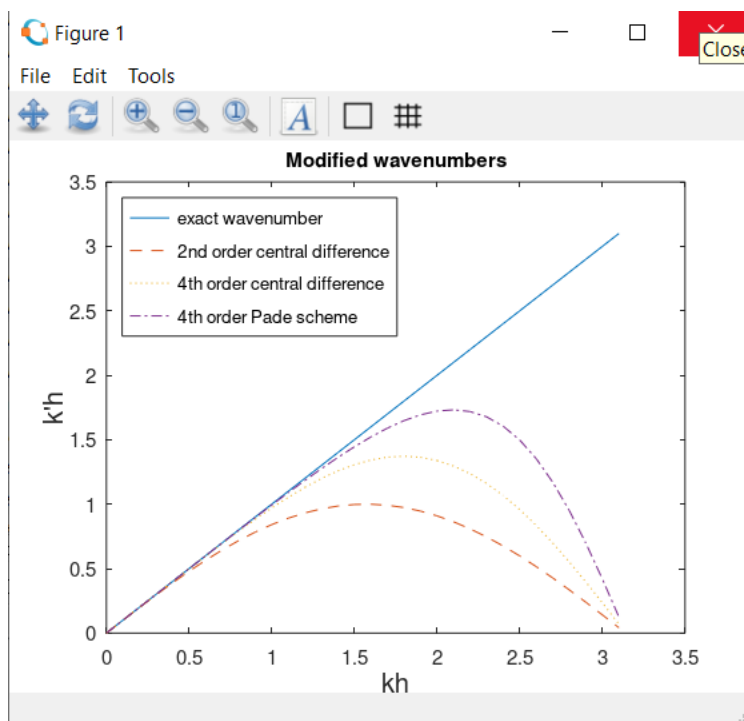
figure(1),clf;
loglog(h,yforward,'o-');
hold on;
loglog(h,ysoc,'rx-');
loglog(h,yfoc,'s-');
set(gca,'FontSize',15);
xlabel("h--grid spacing",'FontSize',20);
ylabel("absolute error",'FontSize',20);
legend('Forward difference','Second-order central difference','Fourth-order central difference','Location','SouthEast')
```



B2.

```
kh = 0:0.1:pi;
khexact = kh;
khsoc = sin(kh);
khfoc = ((8.*sin(kh)).-sin(2.*kh))./6;
khfops = (3.*sin(kh))./(2.+cos(kh));

plot(kh,khexact);
hold on
plot(kh,khsoc, '-');
plot(kh,khfoc, ':');
plot(kh,khfops, '-');
set(gca,'FontSize',15);
xlabel("kh",'FontSize',20);
ylabel("k'h",'FontSize',20);
title("Modified wavenumbers",'FontSize',15);
legend('exact wavenumber', '2nd order central difference', '4th order central difference', '4th order Pade scheme','Location','NorthWest');
```



C2.

```
n = 15;
x = zeros(n,1);
pade = zeros(n,n);
fd=zeros(n,1);
f=zeros(n,1);
h=3/14;

for q=2:n;
    x(q,1)=x((q-1),1)+3/14;
endfor;

#build a pade matrix
for i=1:n;
    for j=1:n;
        if j==i-1;
            pade(i,j)=1;
        endif;
        if j==i;
            pade(i,j)=4;
        endif;
    endfor;
endfor;
```

```

endif;
if j==i+1;
    pade(i,j)=1;
endif;
endfor;
endif;

pade(1,1)=1;
pade(1,2)=2;
pade(n,n-1)=2;
pade(n,n)=1;

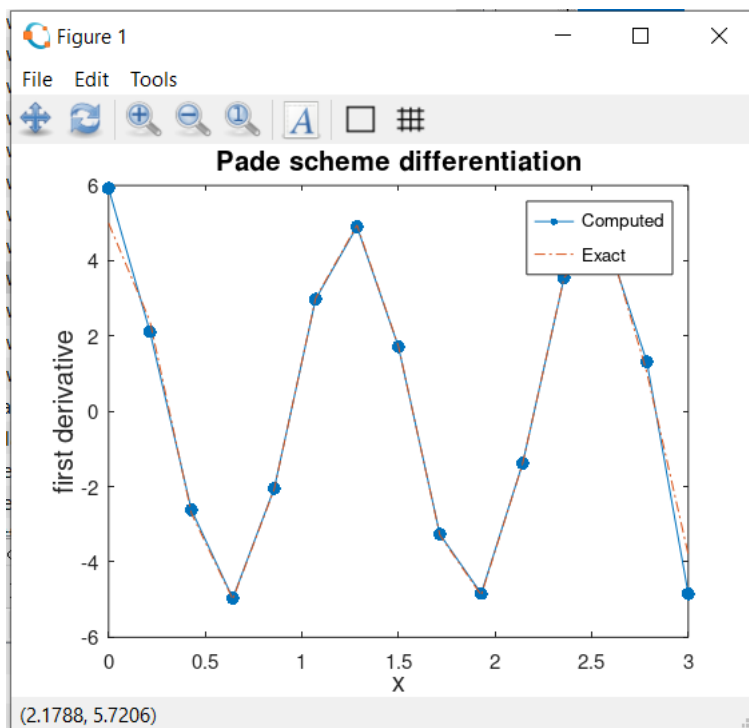
for k=2:n-1;
    f(k,1)=3*(sin(5*x(k+1))-sin(5*x(k-1)));
endfor;

#define the boundaries
f(1,1)=(-5*sin(5*x(1)))/2+2*sin(5*x(2))+sin(5*x(3))/2;
f(n,1)=(5*sin(5*x(15)))/2-2*sin(5*x(14))-sin(5*x(13))/2);

f=f/h;
fd=pade\f;
#fd is found
y=5*cos(x*5);

plot(x,fd,'.-','MarkerSize',15)
hold on;
plot(x,y,'-');
set(gca,'FontSize',15);
xlabel("x",'FontSize',20);
ylabel("first derivative",'FontSize',20);
title("Pade scheme differentiation",'FontSize',20);
legend('Computed','Exact');

```



D4.

```

kh = 0:0.1:pi;
khexact = kh;
khcd = sqrt(-2.*(cos(kh).-1));
khps = sqrt(-12.*(cos(kh).-1)./(cos(kh)+5));

plot(kh, khexact, kh, khcd, kh, khps);

```

```
set(gca, 'FontSize', 15);  
xlabel("kh", 'FontSize', 20);  
ylabel("k'h", 'FontSize', 20);  
title("Modified wavenumbers", 'FontSize', 15);  
legend('exact wavenumber', 'central difference', 'Pade scheme', 'Location', 'NorthWest');
```

