

hw4 B09505021 Numerical Analysis

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B.

```
F1 = @(x) sin(3*x).+3*cos(6*x);
Fd1 = @(x) 3*cos(3*x).-18*sin(6*x);
F2 = @(x) 6*x - x.^2;
Fd2 = @(x) 6 - 2*x;
x = [0:0.01:2*pi];
N = 16;
h = 2*pi/N;
X1 = [0:h:2*pi-h]';
X2 = [h:h:2*pi-h]';
Y1 = F1(X1);
Y2 = F2(X1);

k = [[0:N/2-1]';0;[-N/2+1:-1]'];
CY1 = fft(Y1);
CdY1 = i*k.*CY1;
Yd1 = ifft(CdY1);
CY2 = fft(Y2);
CdY2 = i*k.*CY2;
Yd2 = ifft(CdY2);

n=[1:N-1]';
Yd12 = ((sin(3*h*(n+1))+3*cos(6*h*(n+1)))-(sin(3*h*(n-1))+3*cos(6*h*(n-1))))/(2*h);
Yd22 = ((6*h*(n+1)-(h*(n+1)).^2)-(6*h*(n-1)-(h*(n-1)).^2))/(2*h);

figure(2);

subplot(2,1,2);
plot(x,Fd2(x));
hold on;
plot(X1,Yd2, 'ko');
plot(X2,Yd22, 'ks');
set(gca, 'FontSize', 15);
legend( 'Exact',
        ['FFT, N = ' int2str(N)],...
        'Central Difference');
xlabel('x', 'FontSize', 20);
title('First derivative of B2', 'FontSize', 20);

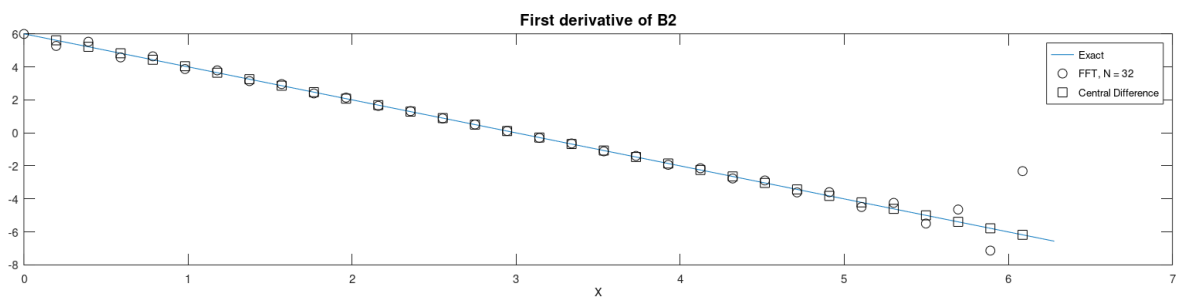
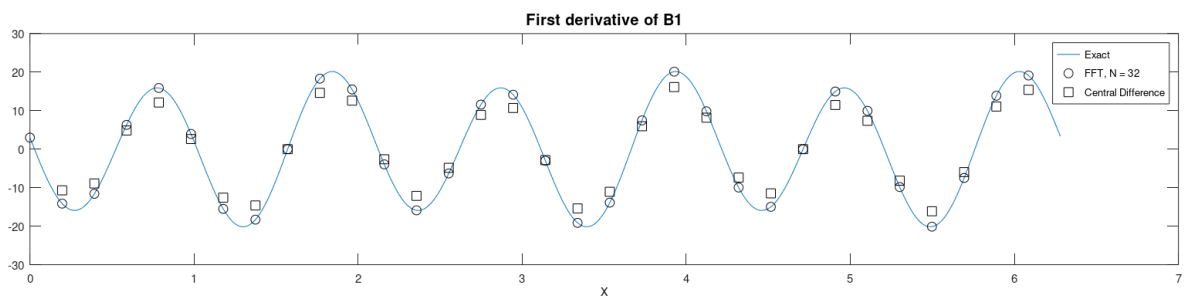
subplot(2,1,1);
plot(x,Fd1(x));
```

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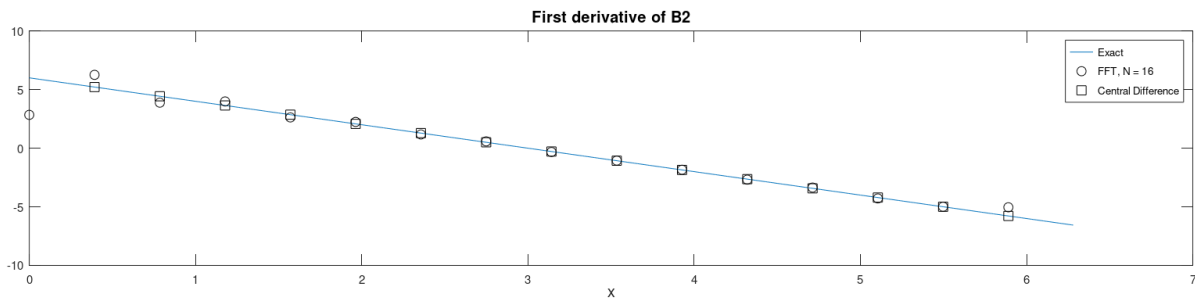
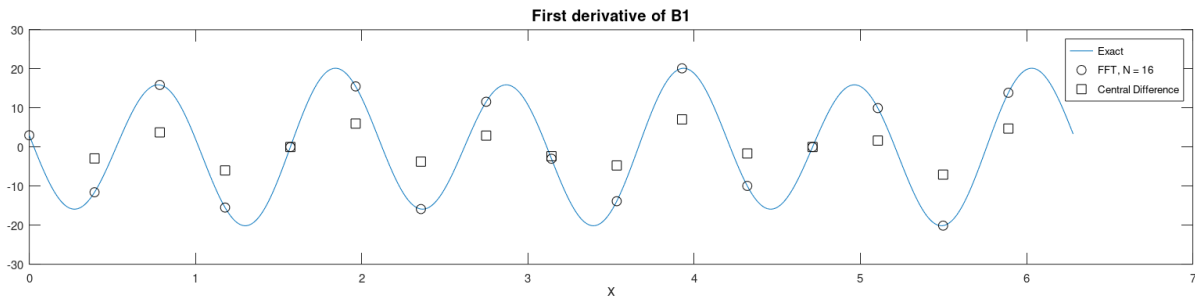
hold on;
plot(X1,Yd1, 'ko');
plot(X2,Yd12, 'ks');
set(gca, 'FontSize', 15);
legend( 'Exact',
        ['FFT, N = ' int2str(N)],...
        'Central Difference');
xlabel('x', 'FontSize', 20);
title('First derivative of B1', 'FontSize', 20);

```

N = 32



N = 16



C.

```

f = @(x) sin(2*x).+0.1*sin(15*x);
g = @(x) sin(2*x).+0.1*cos(15*x);
N = 32;
h = 2*pi/N;
X = [0:h:2*pi-h];
Y = f(X);
G = g(X);

H = Y.*G;
CH1 = fft(H);
H1 = ifft(CH1);

CY = fft(Y);
CG = fft(G);
CYY = [CY(N/2+1:N),CY(1:N/2)];
CGG = [CG(N/2+1:N),CG(1:N/2)];
CHH = zeros(1,N);
for k = [1:N];
    for m = [1:N];
        if and(0<(k-m+(N/2+1)),(k-m+(N/2+1))<N+1);
            CHH(k)+=CYY(m).*CGG(k-m+N/2+1)./N;
        endif;
    endfor;
endfor;
CH2 = [CHH(N/2+1:N),CHH(1:N/2)];
H2 = ifft(CH2);

h3 = @(x) sin(2*x).^2+0.1*sin(2*x).*(sin(15*x).+cos(15*x)).+0.01*sin(15*x).*cos(15*x);

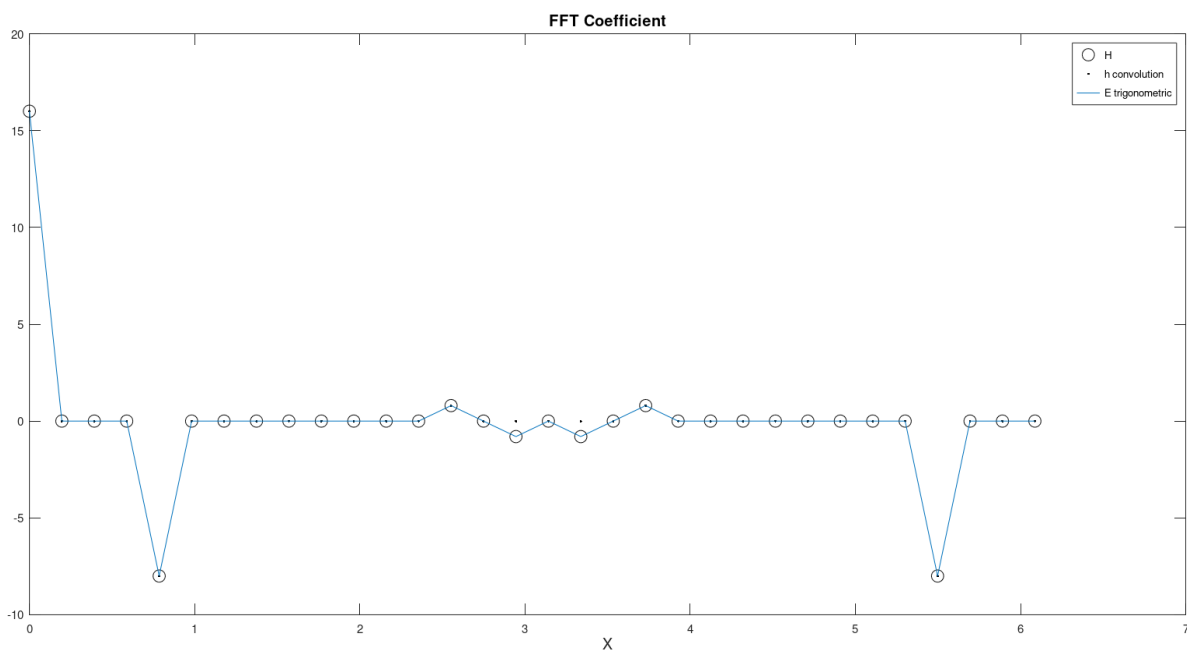
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```

HH = h3(X);
CH3 = fft(HH);
H3 = ifft(CH3);

plot(X,CH1,'ko','markersize',8);
hold on;
plot(X,CH2,'k.','markersize',5);
plot(X,CH3);
set(gca,'FontSize',15);
legend('H','h convolution','E trigonometric');
xlabel('X','FontSize',20);
title('FFT Coefficient','FontSize',20);

```



CH1 =

Columns 1 through 3:

16.0000 + 0i 0.0000 + 0.0000i -0.0000 + 0.0800i

Columns 4 through 6:

0.0000 - 0.0000i -8.0000 + 0.0000i -0.0000 - 0.0000i

Columns 7 through 9:

0.0000 + 0.0000i -0.0000 + 0.0000i -0.0000 - 0.0000i

Columns 10 through 12:

0.0000 - 0.0000i -0.0000 + 0.0000i 0.0000 + 0.0000i

Columns 13 through 15:

-0.0000 - 0.0000i 0.8000 + 0.8000i -0.0000 + 0.0000i

Columns 16 through 18:

-0.8000 + 0.8000i 0.0000 + 0i -0.8000 - 0.8000i

Columns 19 through 21:

$$-0.0000 - 0.0000i \quad 0.8000 - 0.8000i \quad -0.0000 + 0.0000i$$

Columns 22 through 24:

$$0.0000 - 0.0000i \quad -0.0000 - 0.0000i \quad 0.0000 + 0.0000i$$

Columns 25 through 27:

$$-0.0000 + 0.0000i \quad -0.0000 - 0.0000i \quad 0.0000 - 0.0000i$$

Columns 28 through 30:

$$-0.0000 + 0.0000i \quad -8.0000 - 0.0000i \quad 0.0000 + 0.0000i$$

Columns 31 and 32:

$$-0.0000 - 0.0800i \quad 0.0000 - 0.0000i$$

CH2 =

Columns 1 through 3:

16.0000 + 0i 0.0000 + 0.0000i -0.0000 + 0.0000i

Columns 4 through 6:

0.0000 - 0.0000i -8.0000 + 0.0000i -0.0000 - 0.0000i

Columns 7 through 9:

0.0000 + 0.0000i -0.0000 + 0.0000i -0.0000 - 0.0000i

Columns 10 through 12:

0.0000 - 0.0000i -0.0000 + 0.0000i 0.0000 + 0.0000i

Columns 13 through 15:

-0.0000 + 0.0000i 0.8000 + 0.8000i -0.0000 - 0.0000i

Columns 16 through 18:

0.0000 - 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i

Columns 19 through 21:

$$-0.0000 + 0.0000i \quad 0.8000 - 0.8000i \quad -0.0000 - 0.0000i$$

Columns 22 through 24:

$$0.0000 - 0.0000i \quad -0.0000 - 0.0000i \quad 0.0000 + 0.0000i$$

Columns 25 through 27:

$$-0.0000 + 0.0000i \quad -0.0000 - 0.0000i \quad 0.0000 - 0.0000i$$

Columns 28 through 30:

$$-0.0000 + 0.0000i \quad -8.0000 - 0.0000i \quad 0.0000 + 0.0000i$$

Columns 31 and 32:

$$-0.0000 - 0.0000i \quad 0.0000 - 0.0000i$$

CH3 =

Columns 1 through 3:

16.0000 + 0i 0.0000 + 0.0000i -0.0000 + 0.0800i

Columns 4 through 6:

0.0000 - 0.0000i -8.0000 + 0.0000i -0.0000 - 0.0000i

Columns 7 through 9:

0.0000 + 0.0000i -0.0000 + 0.0000i -0.0000 - 0.0000i

Columns 10 through 12:

0.0000 - 0.0000i -0.0000 + 0.0000i 0.0000 + 0.0000i

Columns 13 through 15:

-0.0000 - 0.0000i 0.8000 + 0.8000i -0.0000 + 0.0000i

Columns 16 through 18:

-0.8000 + 0.8000i 0.0000 + 0i -0.8000 - 0.8000i

Columns 19 through 21:

-0.0000 - 0.0000i 0.8000 - 0.8000i -0.0000 + 0.0000i

Columns 22 through 24:

0.0000 - 0.0000i -0.0000 - 0.0000i 0.0000 + 0.0000i

Columns 25 through 27:

-0.0000 + 0.0000i -0.0000 - 0.0000i 0.0000 - 0.0000i

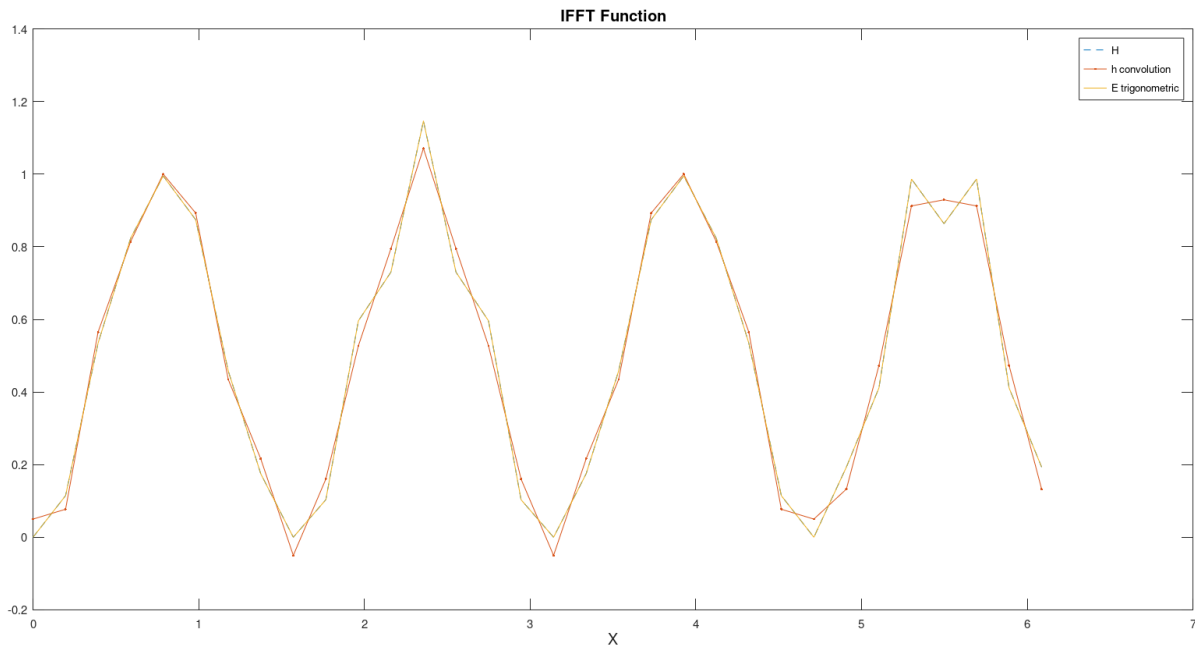
Columns 28 through 30:

-0.0000 + 0.0000i -8.0000 - 0.0000i 0.0000 + 0.0000i

Columns 31 and 32:

-0.0000 - 0.0800i 0.0000 - 0.0000i

```
plot(X,H1,'--','markersize',8);
hold on;
plot(X,H2,'.-','markersize',5);
plot(X,H3);
set(gca,'FontSize',15);
legend('H','h convolution','E trigonometric');
xlabel('X','FontSize',20);
title('IFFT Function','FontSize',20);
```



D.

```

N = 7;
u1 = @(x) 4*(x.^2-x.^4).*exp(-x/2);
u2 = @(x) 2*x.*(x.^3-8*x.^2-x.+4).*exp(-x/2);
u3 = @(x) -(x.^4-16*x.^3+47*x.^2+8*x.-8).*exp(-x/2);
j = [0:N]';
X = cos(j*pi/N);
U = u1(X);
D = zeros(N+1,N+1);
x = [-1:0.01:1];

for j = [1:N+1]
    for k = [1:N+1]
        if j==k
            switch j
                case 1
                    D(j,k) = (2*N^2+1)/6;
                case N+1
                    D(j,k) = -(2*N^2+1)/6;
                otherwise
                    D(j,k) = -X(j)/(2*(1-X(j)^2));
            end;
        else
            if or(j==1,j==N+1) cj = 2;
            else cj = 1;
            end;
            if or(k==1,k==N+1) ck = 2;
            else ck = 1;
            end;
        end;
    end;
end;

```

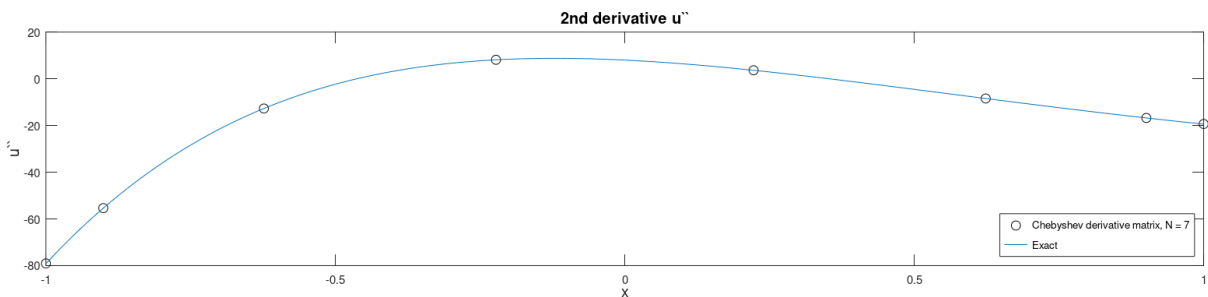
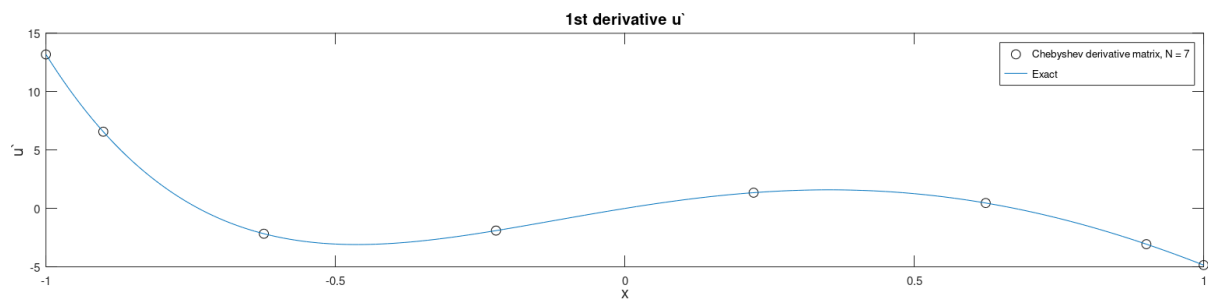
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        D(j,k) = cj*(-1)^(j+k)/(ck*(X(j)-X(k)));
    end;
end;
end;

DU = D*U
DDU = D*D*U

figure(2);
subplot(2,1,1);
plot(X,DU,'ko');
hold on;
plot(x,u2(x));
set(gca,'FontSize',15);
legend(['Chebyshev derivative matrix, N = ' int2str(N)],...
'Exact');
xlabel('x','FontSize',20);
ylabel('u`','FontSize',20);
title('1st derivative u`','FontSize',20);
subplot(2,1,2);
plot(X,DDU,'ko');
hold on;
plot(x,u3(x));
set(gca,'FontSize',15);
legend('Chebyshev derivative matrix, N = 7','Exact','Location','SouthEast');
xlabel('x','FontSize',20);
ylabel('u``','FontSize',20);
title('2nd derivative u``','FontSize',20);

```



```
DU =  
-4.8502  
-3.0599  
0.4658  
1.3497  
-1.8967  
-2.1662  
6.5731  
13.1872
```

```
DDU =  
-19.3404  
-16.7671  
-8.4445  
3.6388  
8.1303  
-12.7193  
-55.3902  
-79.0538
```

```
D =  
16.5000 -20.1957 5.3119 -2.5724 1.6360 -1.2319 1.0521 -0.5000  
5.0489 -2.3929 -3.6039 1.4740 -0.8901 0.6560 -0.5550 0.2630  
-1.3280 3.6039 -0.5100 -2.4940 1.1820 -0.8019 0.6560 -0.3080  
0.6431 -1.4740 2.4940 -0.1171 -2.2470 1.1820 -0.8901 0.4090  
-0.4090 0.8901 -1.1820 2.2470 0.1171 -2.4940 1.4740 -0.6431  
0.3080 -0.6560 0.8019 -1.1820 2.4940 0.5100 -3.6039 1.3280  
-0.2630 0.5550 -0.6560 0.8901 -1.4740 3.6039 2.3929 -5.0489  
0.5000 -1.0521 1.2319 -1.6360 2.5724 -5.3119 20.1957 -16.5000
```

```
>> |
```